Toshi’s epic notes on tree DP

literally based entirely on <https://codeforces.com/blog/entry/20935>

Problem 1: maximum sum of independent set

Given a tree with nodes, in which each node has value . Choose a subset of nodes such that no 2 adjacent nodes are chosen and the sum of of chosen nodes is maximized.

If the tree is a length chain:

Let’s root the tree at node , and let be the answer for the subtree with root .

( is the set of nodes that are children of i)

However, this is a bit hard to implement, and we can use multiple arrays.

Can just use 1 DFS to implement! Time complexity:

Problem 2: diameter of a tree

Given a tree with nodes, calculate its diameter, the length of the longest path between any 2 nodes. (cannot repeat edges or nodes)

Root the tree. the diameter can be 2 possibilities on each node :

1. Start from , goes into its subtree
2. Start from 1 of ’s subtree, go through , ends in another one of its subtrees

Also just use 1 DFS, time complexity:

Problem 3: counting sub trees

Given a tree with nodes and an integer , find the number of different “sub trees” of size . (sub tree is any subset of tree such that every node in it is connected)

Let’s try to count total number of sub trees. (root tree at 1)

The total count is .

Back to our original problem, let’s add another dimension to our DP.

How do we iterate over all possible distinct sequences, a? Another DP, of course!

total time complexity:

Problem 4: expected cost

Given a tree with nodes, in which each node has cost . Steve starts at the root node and navigates to a node he hasn’t visited yet at random. The cost of a path is the sum of costs of nodes visited. Which node should be assigned as root such that the expected cost is minimized?

Again, let’s root the tree at 1.

If we need to find when the tree is rooted at , we also need to find its parent’s contribution.

How do we calculate ? Let be the parent of .

total time complexity:

Problem 5:

Given 2 rooted trees and , find the minimum number of insertions of leaves to any one of the trees to make structurally similar to .

Let’s say is structurally similar too up to node . Let’s say node in has nodes and node in has nodes . We need to create a mapping between and such that will be structurally similar to after insertions. If , we also need to create some subtrees.